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RESERVE BENDING STRENGTH OF STRUTS

(AIRPLANE SECTION, S. & A. BRANCH)

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RESERVE BENDING STRENGTH OF STRUTS.

The object of this investigation is to determine the comparative strength in bending of columns of various materials.

The struts are designed of square spruce, of Specification 10225 round steel tubes, of Specification 10227 round steel tubes and of round duralumin tubes, as columns with various lengths and axial loads.

The resulting columns are then investigated in bending, considering the column to be a simple beam, the maximum allowable concentrated load at the center computed and the results compared.

The columns are then investigated with combined axial and bending loads, each column being assumed to have an axial load equal to 80 per cent of the maximum allowable load, the allowable concentrated bending load at the center computed and the results compared.

CONCLUSIONS.

Spruce and duralumin tube columns designed for the same lengths and axial loads have about the same strength in bending. However, when considered with reference to the weights of the resulting columns, the spruce columns are slightly stronger than the duralumin columns for all lengths and loads covered by this investigation with the exception of the columns designed for the smaller axial loads.

Duralumin tube columns are much stronger in bending than Specification 10227 steel tube columns designed for the same lengths and axial loads. Comparison with reference to their respective weights is even more favorable to the duralumin tube columns.

Columns of Specification 10227 steel tubes weigh a little less, in general, than columns of Specification 10225 steel tubes designed for the same lengths and axial loads. However, the Specification 10227 steel tube columns are much stronger in bending than the Specification 10225 steel tube columns, and the difference is accentuated when the comparison is made with reference to their respective weights.

When compared with reference to their respective strengths with combined axial and bending loads, the square spruce columns in general have a slightly better strength-weight ratio than the duralumin tube columns; the duralumin tube columns have a much better strength-weight ratio than the Specification 10227 steel tube columns; and the Specification 10227 steel tube columns have a much better strength-weight ratio than the Specification 10225 steel tube columns.

Curves were developed, see Figure 1, to show the efficiency of the columns in bending when the bending stresses are combined with axial loads. The spruce columns are slightly more efficient in the Euler range than the columns of other materials, and the Specification 10225 round steel tube columns are the most efficient in the Johnson range. The efficiency of the columns in bending is low excepting for columns with low values of L/ρ, and for the range of this investigation, the efficiencies decrease rapidly as L/ρ approaches the dividing line between the Johnson and Euler ranges.

ASSUMPTIONS.

All columns are assumed to be pin-ended.

The following strength properties are assumed:

Material.	Compressive strength (pounds per square inch).	Modulus of rupture (pounds per square inch).	Modulus of elasticity (pounds per square inch).
Spruce	5,500	10,300	1,600,000
Specification 10227 steel	90,000	110,000	30,000,000
Specification 10225 steel	36,000	55,000	28,000,000
Duralumin	35,000	55,000	10,000,000

The following weights are assumed:

Spruce	27 pounds per cubic foot
Duralumin	175 pounds per cubic foot
Specification 10227 steel	490 pounds per cubic foot
Specification 10225 steel	485 pounds per cubic foot

COMPUTATIONS

Columns of each material are designed with lengths of 10 inches, 30 inches, 60 inches, and 100 inches.

or each length of the various materials columns are designed for axial loads of 1,000 pounds, 5,000 pounds, 10,000 pounds, and 20,000 pounds.

When the allowable load per square inch is greater than 50 per cent of the ultimate compressive strength of the material, Johnson's column formula is used:

$$\frac{P}{A}=f-\frac{f^2}{4C\pi^2E}\times\frac{(L)^2}{(\rho)^2}$$

When the allowable load per square inch is less than 50 per cent of the ultimate compressive strength, Euler's column formula is used:

$$P=\frac{C\pi^2EI}{L^2}$$

As the columns are assumed to be pin-ended, the constant "C" is equal to 1.0 in each case.

In designing the square spruce columns theoretical sizes are used.

In designing the steel and duralumin tube columns the nearest stock size is used, and the concentrated load in bending is corrected for the difference between the stock size and the theoretical size of the column.

Considering the designed columns to be simple beams, the maximum concentrated load at the center is computed by the following formula:

$$\text{The maximum moment}=\frac{PL}{4}=M=\frac{fI}{y}$$

$$P=\frac{4fI}{Ly}$$

where P is the allowable concentrated load at the center.

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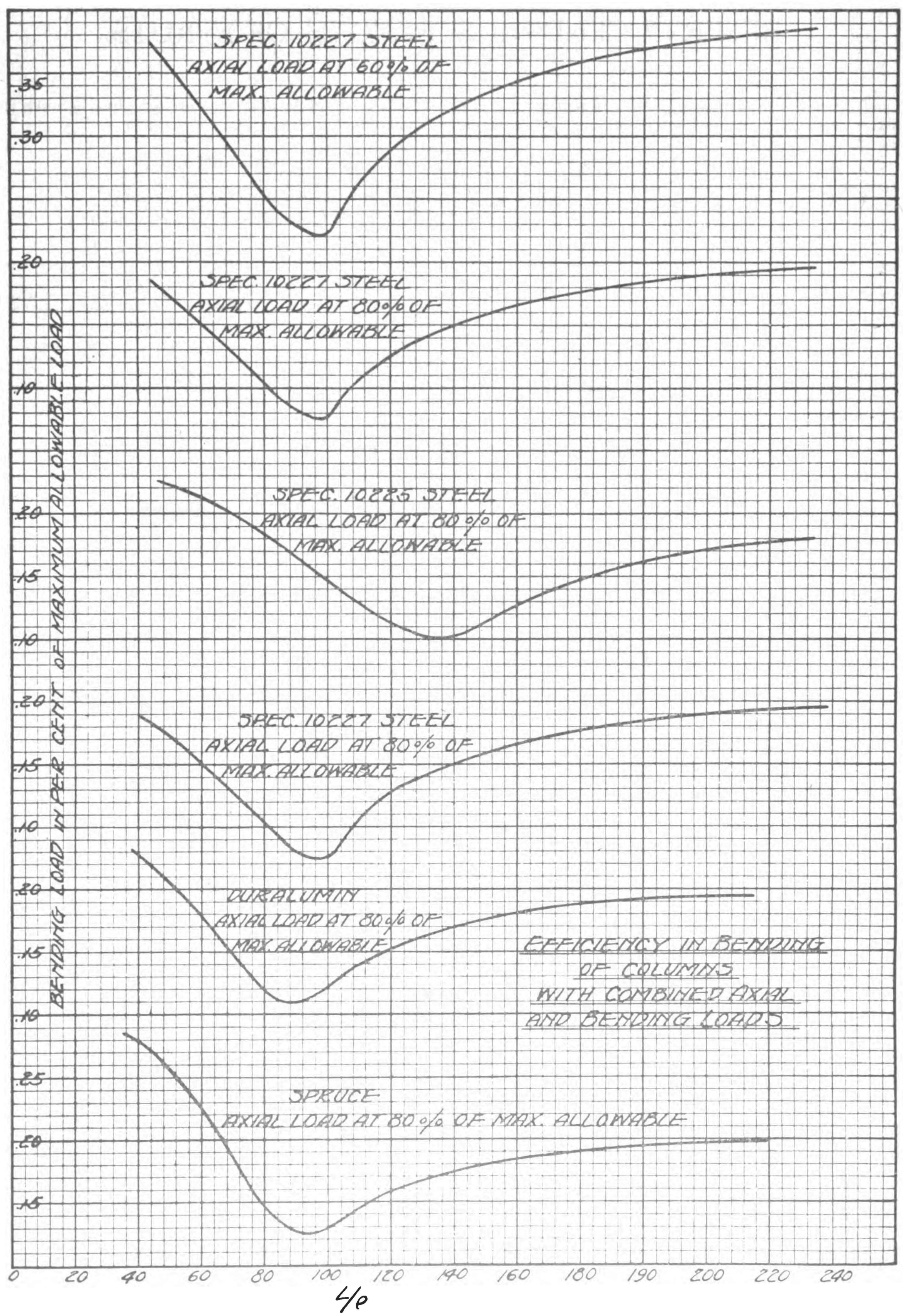
SPEC. 10227 STEEL
AXIAL LOAD AT 60% OF
MAX. ALLOWABLE
SPEC. 10227 STEEL
AXIAL LOAD AT 80% OF
MAX. ALLOWABLE
SPEC. 10225 STEEL
AXIAL LOAD AT 80% OF
MAX. ALLOWABLE
SPEC. 10227 STEEL
AXIAL LOAD AT 80% OF
MAX. ALLOWABLE
DURALUMIN
AXIAL LOAD AT 80% OF
MAX. ALLOWABLE
SPRUCE
AXIAL LOAD AT 80% OF MAX. ALLOWABLE
EFFICIENCY IN BENDING
OF COLUMNS
WITH COMBINED AXIAL
AND BENDING LOADS
BENDING LOAD IN PER CENT OF MAXIMUM ALLOWABLE LOAD
0 20 40 60 80 100 120 140 160 180 190 200 220 240
l/ρ

Fig. 1.

The properties of the designed columns for various materials, lengths, and axial loads are shown in Tables 1, 2, 3, and 4.

The maximum concentrated bending loads for the various columns considered as simple beams are shown in Table 5.

In investigating the columns for a combined axial load and concentrated bending load at the center, the following method is used:

F_c and F_b being assumed, f_c is computed for the column under consideration with an axial load equal to 80 per cent of the maximum allowable load. Substituting in the following formula, the value of f_b is obtained:

$$f_b+f_c=\frac{f_b}{f_b+f_c}\times(F_b-F_c)+F_c$$

$$f_b=\frac{F_b}{2}-f_c+\frac{1}{2}\sqrt{4F_cf_c-4F_bf_c+F_b^2}$$

The maximum moment for a simple beam with a concentrated load at the center $=M=PL/4$.

$$M=M'\times\left(1-\frac{P_1L^2}{10\,EI}\right)=\frac{PL}{4}$$

$$M'=\frac{f_b\,I}{y}$$

$$P=\frac{4f_bI}{Ly}\times\left(1-\frac{P_1L^2}{10\,EI}\right)$$

where P=concentrated bending load at the center

P_1=axial load.

The resulting allowable concentrated bending loads computed in combined compression and bending are to be found in Table 6.

Curves showing the efficiency of the resultant columns in combined compression and bending are shown in Figure 1. The ratio of the allowable bending load in combined compression and bending to the maximum allowable bending load is plotted as ordinates, and the values of L/ρ are plotted as abscissæ.

TABLE 1.—*Square spruce columns.*

Length.	Direct compression.	Width.	Area.	I.	L/ρ	Allowable F_c.	Maximum axial load.	P_b.	P'_b.	Weight.
10	1,000	0.536	0.287	0.0069	64.6	3,500	1,000	105	14.3	0.045
10	5,000	1.008	1.016	.0860	34.4	4,935	5,000	703	171.2	.159
10	10,000	1.388	1.926	.3093	24.9	5,210	10,000	1,840	498.0	.301
10	20,000	1.934	3.740	1.1660	17.9	5,350	20,000	4,960	1,401.0	.585
30	1,000	.909	.827	.0570	114.2	1,208	1,000	172	29.6	.388
30	5,000	1.360	1.850	.2851	76.5	2,715	5,000	575	72.8	.867
30	10,000	1.660	2.756	.6328	62.6	3,620	10,000	1,046	144.8	1.292
30	20,000	2.140	4.580	1.7477	48.5	4,370	20,000	2,250	432.0	2.147
60	1,000	1.286	1.654	.2279	161.3	605	1,000	243	46.5	1.550
60	5,000	1.923	3.698	1.1370	107.8	1,350	5,000	813	135.8	3.466
60	10,000	2.287	5.230	2.2800	90.8	1,905	10,000	1,370	204.5	4.905
60	20,000	2.720	7.398	4.5616	76.5	2,700	20,000	2,300	288.6	6.935
100	1,000	1.660	2.756	.6328	208.7	363	1,000	314	62.2	4.306
100	5,000	2.483	6.165	3.1676	139.4	810	5,000	1,052	193.0	9.633
100	10,000	2.952	8.714	6.3280	117.3	1,146	10,000	1,768	305.6	13.616
100	20,000	3.510	12.320	12.6487	98.8	1,628	20,000	2,950	470.0	19.250

P_b is the maximum allowable concentrated load in center of span considering the column as a simple beam.
P'_b is the allowable concentrated bending load at the center when the axial load on the column is equal to 80 per cent of the maximum allowable load.

TABLE 2.—*Duralumin tube columns.*

Length.	Direct compression.	Outside diameter.	Gauge.	Area.	I.	L/ρ	Allowable F_c.	Maximum axial load.	P_b.	P'_b.	Weight.
10	1,000	½	0.028	0.0415	0.00116	59.8	23,880	992	102	12.3	0.042
10	5,000	⅞	1/16	.1595	.0132	34.7	31,260	4,980	663	129.7	.162
10	10,000	1⅛	3/32	.3037	.0407	27.3	32,680	9,930	1,590	338.4	.307
10	20,000	1⅜	5/32	.5983	.1129	23.0	33,360	19,980	3,614	800.0	.606
30	1,000	⅞	.049	.1272	.0109	102.5	9,390	1,194	182	28.5	.386
30	5,000	1⅛	⅛	.3927	.04985	81.9	13,920	5,470	651	84.0	1.192
30	10,000	1⅜	⅛	.4909	.0968	67.5	20,850	10,220	1,032	110.4	1.490
30	20,000	1¾	5/32	.7824	.2508	53.0	26,250	20,540	2,100	294.0	2.375
60	1,000	1¼	1/16	.2332	.0412	142.7	4,840	1,128	242	44.1	1.417
60	5,000	1½	7/32	8805	.1859	130.4	5,790	5,100	908	158.6	5.350
60	10,000	2	5/32	.9051	.3873	91.7	11,730	10,620	1,420	200.0	5.500
60	20,000	2¼	7/32	1.396	.7283	83.2	14,320	20,000	2,374	307.4	8.475
100	1,000	1½	3/32	.4142	.1008	200.4	2,410	997	296	57.4	4.200
100	5,000	2	7/32	1.2242	.4928	157.4	3,970	4,865	1,083	201.6	12.390
100	10,000	2½	7/32	1.5678	1.0290	123.2	6,470	10,150	1,810	311.5	15.870
100	20,000	3	¼	2.1599	2.0590	102.3	10,060	20,300	3,020	472.0	21.870

P_b is the maximum concentrated load in center of span considering the column as a simple beam.
P'_b is the allowable concentrated bending load at the center when the axial load on the column is equal to 80 per cent of the maximum allowable load.

TABLE 3.—*Steel tube columns, Spec. 10227.*

Length.	Direct compression.	Outside diameter.	Gauge.	Area.	I.	L/ρ.	Allowable F_c.	Maximum axial load.	P_b.	P'_b.	Weight.
10	1,000	11/32	0.028	0.0278	0.00035	89.2	37,200	1,034	89.5	9.5	0.079
10	5,000	3/4	.028	.0635	.00414	39.1	79,500	5,050	486.0	72.9	.180
10	10,000	1⅛	.035	.1199	.0178	25.9	85,400	10,230	1,392.0	251.0	.340
10	20,000	1¼	1/16	.2332	.0412	23.8	86,120	20,080	2,900.0	532.5	.661
30	1,000	5/8	.035	.0649	.00283	143.4	14,310	930	133.0	22.5	.552
30	5,000	1	.049	.1464	.0166	89.1	37,260	5,460	487.0	50.9	1.245
30	10,000	1⅛	1/16	.2086	.0295	79.7	47,500	9,920	769.0	57.3	1.775
30	20,000	1⅛	3/16	.5522	.0631	88.7	37,500	20,740	1,644.0	170.6	4.700
60	1,000	7/8	1/16	.1595	.0132	208.2	6,560	1,085	221.0	42.1	2.714
60	5,000	1⅛	3/16	.5522	.0631	177.4	9,400	5,190	822.0	149.4	9.400
60	10,000	1⅜	3/16	.6995	.1264	141.1	14,870	10,400	1,348.0	222.4	11.900
60	20,000	1¾	5/32	.7824	.2508	106.0	26,340	20,600	2,100.0	280.5	13.300
100	1,000	1¼	.049	.1849	.0334	235.4	5,340	988	235.0	45.9	5.240
100	5,000	1¾	3/32	.4878	.1678	170.6	10,170	4,970	844.0	152.0	13.830
100	10,000	1¾	1/4	1.1781	.3405	186.0	8,560	10,080	1,712.0	317.0	33.400
100	20,000	2¼	7/32	1.3960	.7283	138.4	15,470	21,600	2,852.0	463.0	39.580

P_b is the maximum concentrated load in center of span considering the column as a simple beam.
P'_b is the allowable concentrated bending load at the center when the axial load on the column is equal to 80 per cent of the maximum allowable load.

TABLE 4.—*Steel tube columns, Spec. 10225.*

Length.	Direct compression.	Outside diameter.	Guage.	Area.	I.	L/ρ.	Allowable F_c.	Maximum axial load.	P_b.	P'_b.	Weight.
10	1,000	13/32	0.028	0.03325	0.000597	74.6	29,480	980	65	10.2	0.093
10	5,000	5/8	3/32	.1565	.0057	52.3	32,780	5,130	402	78.8	.439
10	10,000	7/8	1/8	.2945	.02128	37.2	34,380	10,120	1,070	231.5	.827
10	20,000	1⅛	3/16	.5522	.0631	29.6	35,000	19,320	2,470	552.5	1.550
30	1,000	5/8	.049	.0887	.0037	146.7	12,800	1,137	87	11.6	.746
30	5,000	1	1/16	.1841	.0203	90.3	26,450	4,870	298	38.6	1.549
30	10,000	1¼	3/32	.3406	.0573	73.2	29,730	10,120	672	108.3	2.866
30	20,000	1½	5/32	.6596	.1509	62.7	31,400	20,700	1,475	263.5	5.550
60	1,000	7/8	1/16	.1595	.0132	208.3	6,350	1,012	110	19.3	2.686
60	5,000	1¼	1/8	.4418	.07075	150.0	12,280	5,430	415	57.3	7.440
60	10,000	1½	1/8	.5400	.1287	122.7	18,340	9,910	628	65.4	9.090
60	20,000	1¾	3/16	.9204	.2849	107.7	22,400	20,600	1,193	126.3	15.500
100	1,000	1¼	.049	.1849	.0334	235.0	4,980	923	117	21.2	5.185
100	5,000	1½	7/32	.8805	.1859	217.0	5,830	5,130	546	96.2	24.700
100	10,000	2	5/32	.9051	.3873	152.7	11,820	10,700	852	120.0	25.400
100	20,000	2¼	7/32	1.396	.7283	138.3	14,410	20,140	1,425	179.0	39.170

P_b is the maximum concentrated load in center of span considering the column as a simple beam.
P'_b is the allowable concentrated bending load at the center when the axial load on the column is equal to 80 per cent of the maximum allowable load.

TABLE 5.—*Concentrated bending loads.*

Material.	Axial design load.	Length=10 inches.		Length=30 inches.		Length=60 inches.		Length=100 inches.	
		Weight.	P_b.	Weight.	P_b.	Weight.	P_b.	Weight.	P_b.
Square spruce	1,000	0.05	105	0.39	172	1.55	243	4.31	314
Duralumin tubes	1,000	.04	103	.32	152	1.26	214	4.21	297
Spec. 10227 steel tubes	1,000	.08	86.5	.59	143	2.60	203.5	5.30	237.5
Spec. 10225 steel tubes	1,000	.09	66	.66	77	2.66	109	5.62	127
Square spruce	5,000	.16	703	.87	575	3.47	813	9.63	1,052
Duralumin tubes	5,000	.16	665	1.09	595	5.24	891	12.63	1,113
Spec. 10227 steel tubes	5,000	.18	481	1.14	447	9.05	792	13.91	849
Spec. 10225 steel tubes	5,000	.43	392	1.59	306	6.85	382	24.07	532
Square spruce	10,000	.30	1,840	1.29	1,046	4.91	1,370	13.62	1,768
Duralumin tubes	10,000	.31	1,600	1.46	1,010	5.18	1,337	15.62	1,783
Spec. 10227 steel tubes	10,000	.33	1,360	1.79	776	11.43	1,296	33.14	1,698
Spec. 10225 steel tubes	10,000	.82	1,057	2.83	664	9.17	633	23.74	796
Square spruce	20,000	.59	4,960	2.15	2,250	6.94	2,300	19.25	2,950
Duralumin tubes	20,000	.61	3,616	2.31	2,046	8.48	2,374	21.55	2,974
Spec. 10227 steel tubes	20,000	.66	2,888	4.53	1,587	12.91	2,038	36.64	2,640
Spec. 10225 steel tubes	20,000	1.60	2,556	5.36	1,426	15.05	1,158	38.95	1,417

In designing the tube colums for the different lengths and loads the nearest stock size is used, the maximum allowable axial load computed for the size chosen, P_b computed for the size chosen and P_b is then corrected as follows:

Let A=axial load used in design of column.
B=maximum allowable axial load for the tube used.
y=maximum concentrated bending load at center.
x=corrected value of P_b.

$$x=\frac{Ay}{B}$$

The weights of the metal columns are corrected to the weight of the theoretical size in the same manner as the bending loads.

TABLE 6.—*Combined axial and concentrated bending loads.*

Material.	Axial design load.	Actual axial load.	Length=10 inches.		Length=30 inches.		Length=60 inches.		Length=100 inches.	
			Weight.	P'_b.	Weight.	P'_b.	Weight.	P'_b.	Weight.	P'_b.
Square spruce	1,000	800	0.05	14.3	0.39	29.6	1.55	46.5	4.31	62.2
Duralumin tubes	1,000	800	.04	12.4	.32	23.9	1.26	39.1	4.21	57.6
Spec. 10227 steel tubes	1,000	800	.08	9.2	.59	24.2	2.60	38.8	5.30	46.5
Spec. 10225 steel tubes	1,000	800	.09	10.4	.66	10.2	2.66	19.1	5.62	22.9
Square spruce	5,000	4,000	.16	171.2	.87	72.8	3.47	135.8	9.63	193.0
Duralumin tubes	5,000	4,000	.16	130.2	1.09	76.8	5.24	155.4	12.63	207.4
Spec. 10227 steel tubes	5,000	4,000	.18	72.2	1.14	46.7	9.05	143.9	13.91	153.0
Spec. 10225 steel tubes	5,000	4,000	.43	76.8	1.59	39.6	6.85	52.7	24.07	93.8
Square spruce	10,000	8,000	.30	498.0	1.29	144.8	4.91	204.5	13.62	305.6
Duralumin tubes	10,000	8,000	.31	340.5	1.46	108.1	5.18	188.5	15.62	307.2
Spec. 10227 steel tubes	10,000	8,000	.33	245.3	1.79	57.7	11.43	213.8	33.14	314.5
Spec. 10225 steel tubes	10,000	8,000	.82	229.0	2.83	107.1	9.17	66.0	23.74	112.2
Square spruce	20,000	16,000	.59	1,401.0	2.15	432.0	6.94	288.6	19.25	470.0
Duralumin tubes	20,000	16,000	.61	801.0	2.31	286.2	8.48	307.4	21.55	465.0
Spec. 10227 steel tubes	20,000	16,000	.66	530.0	4.53	164.5	12.91	272.3	36.64	428.5
Spec. 10225 steel tubes	20,000	16,000	1.60	572.0	5.36	254.5	15.05	122.7	38.95	167.2

P'_b is the maximum allowab'e concentrated bending load at the center of the column when the column is sustaining an axial load equal to 80 per cent of the maximum allowab!e load. As the nearest stock size was used in the design of the tube columns, the values of P'_b have been corrected by proportion to the design axial load, as shown in note to Tab'e 5.

The weights of the metal columns are corrected to the weight of the theoretical size by the same method as the bending loads.

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